Control of Processes with Noise and Time Delays

A stable Kalman filter predictor (KFP) is developed which generates minimum variance estimates of the future outputs $\{y(t+i|t), i = 1, \ldots, d\}$ of stochastic, single-input/single-output processes with time delay, d. The predicted outputs are used for time delay compensation and in the design of a predictive feedback controller. An innovation model analysis is used to convert the state space formulation to transfer function form and to show the relationship between the KFP, the Smith predictor, and the internal model controller. A modified KFP includes a disturbance model, and eliminates offset due to deterministic disturbances (e.g., steps) and modeling errors. Simulation results show that the modified KFP also predicts the disturbances and gives significantly better performance than the Smith predictor, particularly in the presence of process and measurement noise.

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Introduction

Some of the most common problems in process control applications arise because of time delays, noise, and sustained disturbances. The objective of this work was therefore to develop a feedback control system that would:

1. Compensate for time delays
2. Provide optimal estimates of the required process values in the presence of noise
3. Eliminate any estimation problems due to offset caused by sustained disturbances and/or modeling errors

Time delay compensation requires the use of a process model. However, instead of using a transfer function or input/output model it was decided to use a state space approach because that is convenient for prediction and permits formal separation and rigorous treatment of process noise, measurement noise, and structured disturbances. The development presented in this paper therefore has four main steps:

1. Development of a suitable state space process model
2. Formulation of the Kalman filter predictor (KFP)
3. Innovation model analysis to convert the KFP to an equivalent transfer function form for comparison
4. Simulation results to illustrate the performance of the KFP used in conjunction with PID and predictive controllers

Time delay compensation in process applications began with the Smith predictor (Smith, 1957, 1959); at about the same time Kalman (1960) developed the Kalman filter. Both are now standard textbook material, so a general review of the literature in these areas is omitted, but specific references are inserted at appropriate places in the following development. The references most relevant to this paper are those by Bialkowski (1978, 1983), who used a steady state Kalman filter in conjunction with a standard PID controller plus a LQG controller to control a pulp and paper process with time delays. This paper extends Bialkowski's approach to a general time-varying formulation, provides the necessary theoretical background, formally shows the equivalence of the KFP to alternative approaches, and includes the development and evaluation of a predictive feedback controller.

Formulation of the Kalman Filter Predictor

The first step in the formulation of the KFP is the development of a suitable state space model of the process. This is followed by sections dealing with the formulation of the KFP and by example applications of time delay compensation.

Process model

The key step in the formulation of the Kalman filter predictor (KFP) is the selection of the state space model used to represent the overall process. As shown in Figure 1, the process is assumed to have an input u, output y, time delay d, plus process noise w and measurement noise v. For modeling purposes the process is divided into two parts: first, the process without delays (except the unity delay due to the zero-order hold) and second, the delay

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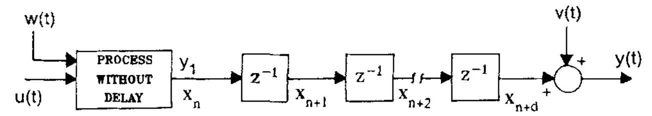

Figure 1. Stochastic, SISO process with delay d.

part of the process, which is assumed to consist of d first-order delays in series. The two parts of the process are modeled separately and then augmented into a single state space model that is the basis for the Kalman filter. Deterministic (i.e., nonstationary) disturbances such as sustained steps are considered later in a separate section.

Model of Overall Process. Assume that the process of interest can be adequately modeled by an nth-order ARMA representation.

$$y(t) = A^{-1}(q^{-1})B(q^{-1})u(t) \tag{1}$$

where

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_nq^{-n} \tag{2}$$

$$B(q^{-1}) = q^{-(1+d)}(b_1 + b_2q^{-1} + \cdots + b_nq^{-n+1}) \tag{3}$$

and d is the time delay in the process, excluding the discretization delay.

Model of Process without Delays. The ARMA representation for the process without time delays is given by

$$y_1(t) = q^{-1}A^{-1}(q^{-1})B'(q^{-1})u(t) \tag{4}$$

where

$$B'(q^{-1}) = b_1 + b_2q^{-1} + \cdots + b_nq^{-n+1} \tag{5}$$

The objective now is to find a suitable state space formulation that is equivalent to the above ARMA model and satisfies the following properties:

1. Observability
2. Contains the desired predicted output $\{y(t+i|t), i = 1 \ldots d\}$ as states
3. The only nonzero, nonunity parameters are equal to the ARMA parameters in Eq. 1

The final state space model for the process without delays is defined by the following state space equations:

$$\mathbf{x}(t+1) = \Phi\mathbf{x}(t) + \Lambda u(t) + \Gamma w(t) \tag{6}$$

$$y(t) = \Theta\mathbf{x}(t) + v(t) \tag{7}$$

where

$$\Phi = \Phi_1 = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & -a_n \\ 1 & 0 & \cdot & \cdot & \cdot & -a_{n-1} \\ \vdots & & & & & \\ 0 & 0 & \cdot & 1 & 0 & -a_2 \\ 0 & 0 & \cdot & 0 & 1 & -a_1 \end{bmatrix}_{n\times n} \qquad \Lambda = \Lambda_1 = \begin{bmatrix} b_n \\ b_{n-1} \\ \vdots \\ b_2 \\ b_1 \end{bmatrix}_{n\times 1}$$

$$\Gamma = \Gamma_1 = \begin{bmatrix} \gamma_1 \\ \cdots \\ \gamma_n \end{bmatrix}_{n\times 1}$$

$$\Theta = \Theta_1 = [0 \quad 0 \quad \cdots \quad 0 \quad 1]_{1\times n}$$

and $w(t)$ and $v(t)$ are assumed to be uncorrelated random sequences with zero mean. The matrix Γ is defined such that process noise can be added to all n states. The noise covariances are defined by:

$$E\{w(t)w(t)^T\} = R_w$$
$$E\{v(t)v(t)^T\} = R_v$$

Note that the n (order of the ARMA process) states define the process without delays and $x_n = y_1$ in Figure 1.

Model of Process Delays. The process time delay is assumed to be a known integer multiple of the sampling time and is represented by d unit time delays in series. As implied by Figure 1, this is easily put into state space form once a single delay can be represented by:

$$x_i(t+1) = x_{i-1}(t)$$

Thus the d delays in Figure 1 can be represented by Eqs. 6 and 7 with $u(t) = x_n(t)$ from the preceding process model, $y(t) = y(t)$ in Figure 1, $x(t) = [x_{n+1}(t), \ldots, x_{n+d}(t)]$ in Figure 1, and

$$\Phi = \Phi_2 = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \vdots & \vdots & & & & \vdots \\ 0 & 0 & \cdot & \cdot & 1 & 0 \end{bmatrix}_{d\times d} = \begin{bmatrix} 0 \\ e_1^T \\ e_2^T \\ \\ e_{d-1}^T \end{bmatrix}_{d\times d}$$

$$\Lambda = \Lambda_2 = [1 \quad 0 \quad \ldots \quad 0]^T_{1\times d}$$

$$e_j^T = [0 \quad 0 \quad \cdots \quad 1 \quad \cdots \quad 0]_{1\times d}$$

$$\Theta = \Theta_2 = [0 \quad 0 \quad \cdots \quad 0 \quad 1]_{1\times d}$$

$$\Gamma = \Gamma_2 = 0$$

Note that x_{n+1} to x_{n+d} are simply delayed values of $x_n = y_1$ and that x_{n+d} is the actual output of the overall process (without noise).

State Space Model of Overall Process. Assuming that only states of the process without time delays are influenced by the process noise, and also introducing measurement noise only at the output, x_{n+d}, the above two systems can be augmented to give a state space formulation, which represents the overall single-input/single-output (SISO) process with time delays. The augmented state space model has the standard form defined by Eqs. 6 and 7 with:

$$\Phi = \begin{bmatrix} \Phi_1 & 0 \\ \Phi_X & \Phi_2 \end{bmatrix}_{(n+d)\times(n+d)} \qquad \Lambda = \begin{bmatrix} \Lambda_1 \\ 0 \end{bmatrix}_{(n+d)\times 1} \qquad \Gamma = \begin{bmatrix} \Gamma_1 \\ 0 \end{bmatrix}_{(n+d)\times 1}$$

$$\Phi_X = \begin{bmatrix} \Theta_1 \\ 0 \end{bmatrix}_{d\times n} \qquad \Theta = [0 \quad \Theta_2]_{1\times(n+d)}$$

Note that the state vector is $x(t) = [x_1, x_2, \ldots, x_n, x_{n+1}, \ldots, x_{n+d}]^T$ and that x_{n+1} to x_{n+d} are simply delayed values of $x_n = y_1$ and that x_{n+d} is the actual output of the overall process (without noise).

In the simulated examples presented later it is assumed for simplicity that the elements γ_1 to γ_n of Γ_1 are equal to unity, and the KFP can be tuned by adjusting the ratio R_w/R_v. The dimension of the state vector is a function of the order of the process and of the time delay, and hence depends on the sampling time used for the discretization. The state transition matrix Φ is singular but this can be accommodated by the Kalman filter implementation discussed below.

Kalman filter predictor

The states $x_n(t)$ to $x_{n+d-1}(t)$ of the state space formulation, Eqs. 6 and 7, are the future outputs $y(t + d)$ to $y(t + 1)$, respectively. Thus, estimating the states of the augmented process at time t is equivalent to predicting the future outputs. In principle any state estimator can be used to estimate these states. However, since it is assumed that both process and measurement noise are zero-mean, uncorrelated random sequences, a Kalman filter is optimal.

The Kalman filter (KF) was first presented by Kalman (1960) and Kalman and Bucy (1961). The theory, properties, and implementation of the KF are discussed in a number of references, including Astrom (1970), Astrom and Wittenmark (1984), and Goodwin and Sin (1984). The predictor scheme, based on the state space formulation given by Eqs. 6 and 7 and the Kalman filter, is called the Kalman filter predictor (KFP) and can be implemented using either a steady state Kalman filter or a time-varying formulation. The time-varying Kalman filter algorithm used in the KFP, is a two-step algorithm (Franklin and Powell, 1980) in which the states and the covariances are updated between measurements. The time-varying Kalman filter algorithm is as follows.

a. *Gain Calculation:*

$$L(t) = M(t)\Theta^T[\Theta M(t)\Theta^T + R_2]^{-1} \tag{8}$$

where $R_2 = R_v$.

b. *Measurement Update* (at the time of measurement):

State Update:

$$\hat{x}(t) = \bar{x}(t) + L(t)\,[y(t) - \Theta\bar{x}(t)] \tag{9}$$

Covariance Update:

$$P(t) = M(t) - L(t)\Theta M(t) \tag{10}$$

c. *Time Update* (between measurements):

State Update:

$$\bar{x}(t + 1) = \Phi\hat{x}(t) + \Lambda u(t) \tag{11}$$

Covariance Update:

$$M(t + 1) = \Phi P(t)\Phi^T + R_1 \tag{12}$$

where $R_1 = \Gamma R_w \Gamma^T$.

The steady state Kalman filter predictor can be implemented by calculating the steady state Kalman gains using the methods

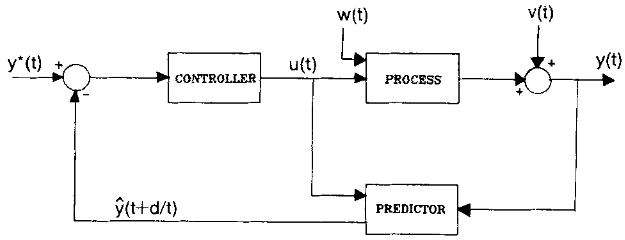

Figure 2. Feedback control using a predictor for time delay compensation.

proposed by Pappas et al. (1980), which are computationally and numerically efficient.

If the process model given by Eq. 4 is stable, it can be proved that the state space model of the KFP is observable and stabilizable (Walgama, 1986). These two properties guarantee that the steady state Kalman filter predictor is a stable predictor, that is, that the steady state filter transition matrix $\bar{\Phi} = (\Phi - L\Theta\Phi)$ has its eigenvalues inside the unit circle, and the time-varying Kalman filter predictor converges to a stable, steady state Kalman filter predictor as $t \rightarrow \infty$.

Feedback control using the KFP

The predicted future outputs, and if desired the other estimated states of the KFP, can be employed in any type of controller design. Bialkowski (1978) used a steady state KFP with PID and LQG controllers. The conventional PID control algorithm can be employed with the KFP as shown in Figure 2. The tracking error $e(t)$ is calculated using the predicted output $\hat{y}(t + d|t)$ rather than the measured output $y(t)$, and is given by,

$$e(t) = y^*(t) - \hat{y}(t + d|t) \tag{13}$$

If the future desired output, $y^*(t + d)$ is known d steps ahead, it can be used to calculate the error instead of $y^*(t)$. [The PID controller parameters are tuned for the model without delays (Figure 6 and Eq. 14), rather than for the actual process.]

Figure 3 shows the performance of a PI feedback control scheme using the KFP for a first-order process, with process and measurement noise. The KFP can be tuned using the ratio of the covariances R_w/R_v to give minimum variance control performance as indicated by the bottom curve in Figure 4. (The difference between the KFP and the Smith predictor is discussed further in a later section).

Figure 5 shows the performance of the KFP in the presence of deterministic disturbances. Even though a PI controller is used,

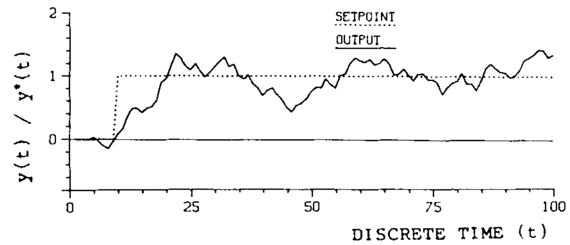

Figure 3. KFP plus PID control of a first-order stochastic process with a step change in set point.

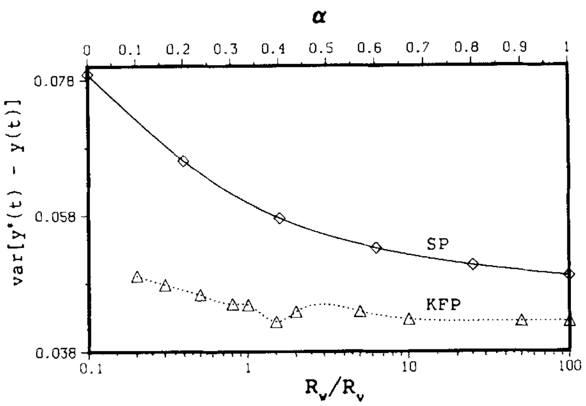

Figure 4. Output error variance for KFP and SP using PI control.
R_w/R_v – ratio of covariance
α = exponential filter constant

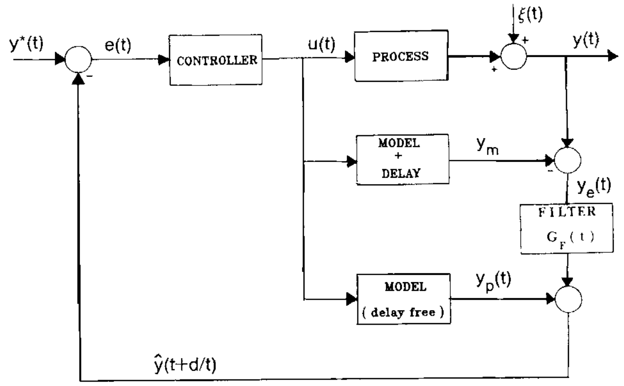

Figure 6. Transfer function representation of KFP.

there is an offset in the output when sustained disturbances enter the system. This offset can be reduced, but not eliminated, by increasing the covariance ratio R_w/R_v. The difficulties encountered by the KFP in the presence of deterministic disturbances are investigated and solved in the next section by deriving the equivalent input/output (transfer function) model of the KFP.

Innovation Model Analysis for KFP

The KFP gives the minimum variance estimates of the states, that is, predicted process outputs, but its structure and internal behavior are not apparent from the state space formulation. The internal operation of the KFP can be better understood by deriving the transfer function model of the Kalman filter. Owing to the large dimensions of the matrices involved, a general derivation is cumbersome. Hence the innovation model concept due to Kailath (1970) was used as outlined in Appendix A, to transform the state space KFP formulation into an equivalent transfer function form. (Appendices are available as supplementary material from AIChE or in Walgama, 1986.) The end result is:

$$\hat{y}(t + d|t) = G_p(q^{-1})u(t) + G_F(t, q^{-1})[y(t) - G_m(q^{-1})u(t)] \quad (14)$$

or,

$$\hat{y}(t + d|t) = y_p(t) + G_F(t, q^{-1})[y(t) - y_m(t)] \quad (15)$$

A block diagram of the internal structure of the KFP, based on Eq. 15, is shown in Figure 6; this permits a simple interpretation of the KFP.

Comments on Filter G_F. In the KFP, Figure 6, the error $y_e(t)$, between the actual output y and the model output y_m is filtered by the time-varying filter $G_F(t, q^{-1})$, before being added to the output y_p of the model G_p, to obtain the predicted output $\hat{y}(t + d|t)$. $y_e(t)$ represents the noise, disturbances, plus any unmodeled dynamics present in the system. The time-varying nature of the filter $G_F(t, q^{-1})$ is due to the time-varying Kalman gains. This time-varying filter would asymptotically converge to a steady state filter if the Kalman gains converge to their steady state values.

The stability of the KFP, that is, the boundedness of the prediction $\hat{y}(t + d|t)$ can be interpreted as the stability of the filter $G_F(t, q^{-1})$ if the model G_m is a stable system. Walgama (1986, Appendices A and B) proved that $G_F(q^{-1})$ is a stable filter.

KFP vs. Internal Model Control (IMC). The IMC by Garcia and Morari (1982) has the same structure as shown in Figure 6 but with different definitions for G_p and G_F. In this work the design of the filter is determined by the Kalman filter approach and results in minimum variance estimates of the process outputs. Similarly, G_p is simply the model of the process without

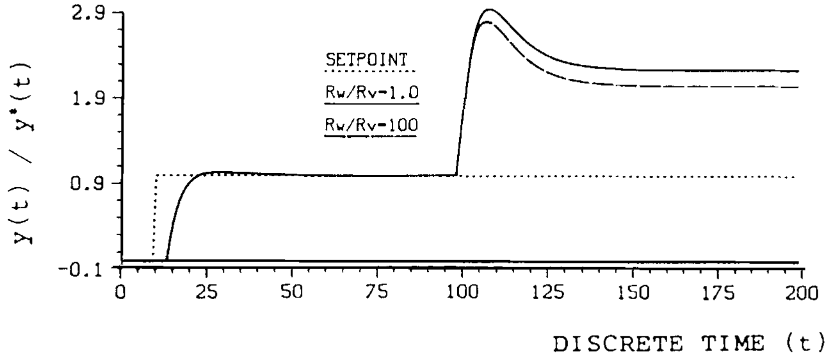

Figure 5. KFP plus PI control of a first-order process.
Step set point change at t = 10, step disturbance at t = 100

delays. In IMC the filter is designed on a more *ad hoc* basis and used to trade off performance vs. robustness.

Kalman Filter Predictor vs. Smith Predictor (SP). It is easy to show that the deterministic (noise-free) predictor developed by Smith can also be represented by Figure 6 if the filter transfer function G_F is set to unity. Thus it is interesting to compare the KFP and the SP more formally. To compare the behavior of the KFP and SP, the closed-loop equation and the predictor equations are derived for both of these schemes. For the SP (omitting the q^{-1} for clarity)

$$\hat{y}(t+d|t) = G_p u(t) + [y(t) - G_m u(t)] \tag{16}$$

and the closed-loop equation for an error-driven feedback control system with output disturbances $\xi(t)$ is,

$$y(t) = \frac{GG_c}{1 + G_c G_p + G_c(G - G_m)} y^*(t) - \frac{GG_c}{1 + G_c G_p + G_c(G - G_m)} \xi(t) + \xi(t) \tag{17}$$

For the KFP, the predictor equation is given by Eq. 14,

$$\hat{y}(t+d|t) = G_p u(t) + G_F(t)[y(t) - G_m u(t)] \tag{18}$$

and the closed-loop equation is given by,

$$y(t) = \frac{GG_c}{1 + G_c G_p + G_c(G - G_m)G_F(t)} y^*(t) - \frac{G_F(t) G_c G}{1 + G_c G_p + G_c(G - G_m)G_F(t)} \xi(t) + \xi(t) \tag{19}$$

Remark 1. The SP is designed to work with deterministic processes. When the process is corrupted by process and/or measurement noise, the SP handles it by simply adding the error, $y_e(t)$, to $y_p(t)$ to obtain the prediction $\hat{y}(t+d|t)$. The error $y_e(t)$ represents the noise present in the process measurement, disturbances, plus unmodeled dynamics in the SP. This could result in unsatisfactory control. The performance of the SP can be improved for applications with stochastic measurement noise by introducing a filter (e.g., the familiar exponential filter) in the same location as G_F in the KFP. However the KFP is the preferred method for applications with Gaussian noise since it is based on a sound theoretical formulation rather than heuristic arguments and gives minimum variance estimates.

Remark 2. When there is no mismatch between the process and the model, and the system is free of measurement noise, process noise, and disturbances, then $G_m = G$ and $\xi(t) = 0$. Therefore for both SP and KFP

$$\hat{y}(t+d|t) = G_p(q^{-1})u(t) \tag{20}$$

and

$$y(t) = \frac{GG_c}{1 + G_c G_p} y^*(t) \tag{21}$$

Thus under perfect modeling and noise-free conditions the SP and KFP give the same output prediction and output control, assuming that they both use the same controller, which is tuned to G_p. (When the KFP is implemented using the KF algorithm, G_p is not explicitly present in the scheme. The tuning of G_c to control G_p using the KFP can be done by setting $R_w/R_v = 0$, which makes $G_F = 0$, and is appropriate for the noise-free case.)

Remark 3. In the presence of process and measurement noise [$\xi(t) \neq 0$], but under perfect modeling, the prediction and the output control of the SP and KFP are given by the following equations.

For the SP, from Eqs. 16 and 17,

$$\hat{y}(t+d|t) = G_p u(t) + [y(t) - G_m u(t)] \tag{22}$$

$$y(t) = \frac{GG_c}{1 + G_c G_p} y^*(t) - \frac{GG_c}{1 + G_c G_p} \xi(t) + \xi(t) \tag{23}$$

For the KFP, from Eqs. 18 and 19,

$$\hat{y}(t+d|t) = G_p u(t) + G_F[y(t) - G_m u(t)] \tag{24}$$

$$y(t) = \frac{GG_c}{1 + G_c G_p} y^*(t) - \frac{G_c G G_F}{1 + G_c G_p} \xi(t) + \xi(t) \tag{25}$$

It is clear from Eqs. 23 and 25 that the closed-loop characteristic equations for set point tracking are the same for both SP and KFP. But with noise $\xi(t)$, the roots of the closed-loop characteristic equation for the KFP contain the poles of the filter G_F.

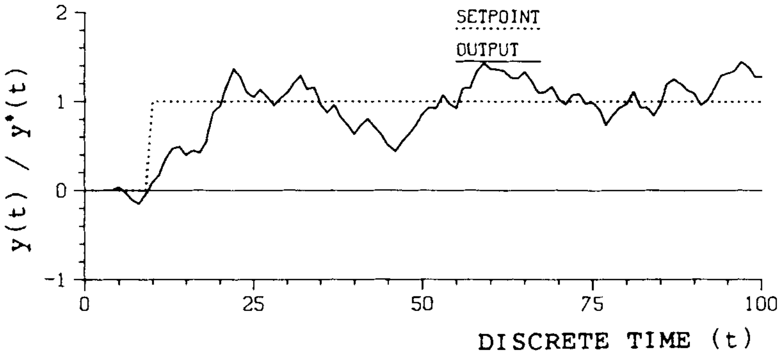

Figure 7. SP plus PI control of a first-order stochastic process.
See Figures 3 and 4

Since G_F was proved to be stable, the introduction of the filter G_F does not affect the stability of the closed-loop system. Note that with the KFP G_c can be tuned to provide the desired response for set point changes and G_F (e.g., by adjusting R_w/R_v in the KF) can be tuned independently to provide the desired regulatory response.

Example. The performance of the KFP and the Smith predictor, using the same PI controller, is shown in Figures 3 and 7, respectively. Figure 4 shows the variance of the output error under PI control using the KFP and the SP. Clearly the KFP gives lower error variance than the SP. The introduction of an exponential filter will smooth the response of the SP but does not improve the error variance, as shown in Figure 4 (setting $\alpha = 1$ means $G_F = 1$).

The KFP will reduce to an SP if $G_F = 1$, but it is difficult to say under which conditions $G_F = 1$. Even under steady state conditions, there is no guarantee that $G_{F_{ss}} = 1$. This problem is examined in the next section.

KFP Modified for Deterministic Disturbances

A better understanding of the performance of the KFP in the presence of deterministic disturbances can be achieved by a further expansion of the transfer function configuration of the KFP. For convenience, consider only the steady state Kalman filter. From the innovation model analysis of the KFP in Appendix B (supplementary material) it can be shown that the structure of the KFP in Figure 6 can be expanded to that shown in Figure 8 (disregard the broken-line elements at this stage).

To compensate for the disturbances, the full information regarding the disturbances has to be transmitted and included in the prediction $\hat{y}(t+d|t)$. That is, $\omega'(t)$ in Figure 8 should be equal to $\xi(t)$ if perfect modeling is assumed. The steady state value of $\omega'(t)$ is,

$$\omega'_{ss} = \frac{\sum_{r=1}^{n} L_r}{\sum_{r=1}^{n} a_r + \sum_{r=1}^{n+d-1} h_r} \xi_{ss} \tag{26}$$

From Eq. 26 it is clear that the KFP configuration does not guarantee that $\omega'_{ss} = \xi_{ss}$. That is, it does not guarantee that the KFP will pass all the information about the deterministic disturbances to the predicted output $\hat{y}(t+d|t)$. This results in a bias in the predicted output $\hat{y}(t+d|t)$ and hence in the control or regulation of $y(t)$ even if G_c includes integral action. This bias in the predictor can be attributed to the proportional plus derivative (PD) estimation nature of the KFP, Figure 8, and the effect of the bias on the controlled response is illustrated by Figure 5.

Formulation of modified KFP

Based upon the interpretation given in the above section regarding the bias problem in the KFP, an intuitive approach to overcome this problem is to change the PD estimation of the KFP to a PID estimation scheme by introducing a set of integrators in parallel with the PD controllers of G_{F1} and G_{F2}, as shown by the broken lines in Figure 8. Assume for simplicity of explanation that the disturbance ξ is a step of magnitude ξ_{ss}. Then the integral feedback action added to the KFP makes the steady state value of $\bar{\omega}(t)$ go to zero, by making the output of the integrator go to ξ_{ss}. At steady state $\omega'(t)$ will also reach zero, and

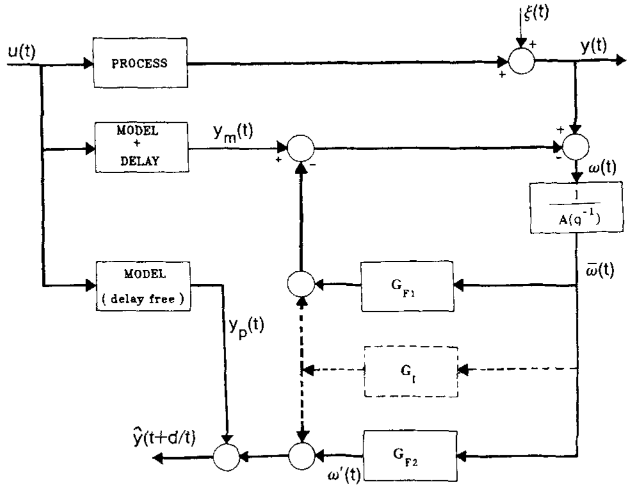

Figure 8. Extended transfer function representation of MKFP.

-----Integrators added to remove offset
$G_{F1} = m_1$: P + D controllers
$G_{F2} = m_2$: P + D controllers
$G_I = m_2$: Integrators

since the output of the integrator is included in the predicted output $\hat{y}(t+d|t)$, the bias is removed. This *ad hoc* way of modifying the KFP can be achieved more formally by modifying the state space formulation as suggested by Balchan et al. (1971, 1973), and as used by Bialkowski (1983) in his steady state KFP. When the process noise is not white noise, the state space model of the process in Eqs. 6 and 7 can be augmented by the states corresponding to the noise dynamics. By interpreting the process noise, x_p, as a random signal that changes at random time instants, by random step sizes, the dynamics of the disturbances can be represented by integrated white noise (Balchan et al., 1971, 1973; Tuffs and Clarke, 1985), as follows,

$$x_p(t) = \frac{w(t)}{\Delta} \tag{27}$$

where

$\Delta = 1 - q^{-1}$ differencing operator
$w(t)$ = random signal, generally zero but attains values p_i at arbitrary time instants i
$x_p(t)$ = series of steps of height p_i at times i

The augmented state space model, Eqs. 6 and 7, can then be augmented with an additional state having an integrator to represent noise dynamics.

$$x(t+1) = \begin{bmatrix} 1 & 0 & 0 \\ \Gamma_1 & \Phi_1 & 0 \\ 0 & \Phi_x & \Phi_2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \Lambda \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w(t) \tag{28}$$

$$y(t) = [0:0:\Theta_2]x(t) + v(t) \tag{29}$$

where

$$x(t) = [x_p, x_1, x_2, \ldots, x_n, x_{n+1}, \ldots, x_{n+d}]$$

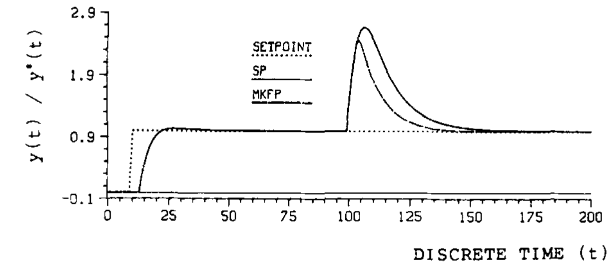

Figure 9. MKFP and SP plus PI control of a first-order process.

If the elements of Γ_1 are unity then this formulation is the same as the previous augmented state space model except that $w(t)/\Delta$ is added to each state instead of $w(t)$. This makes it possible to handle nonzero mean (i.e., nonstationary) disturbances, $\xi(t)$.

As shown in Appendix C (supplementary material), an innovation model analysis of the KFP based on Eqs. 28 and 29 leads to the introduction of extra integrators as shown by the broken lines in Figure 8. This modified KFP (MKFP) can also be represented by the simpler block diagram in Figure 6. The only difference between the KFP and the MKFP is the definition of the filter transfer function, G_F.

Modified KFP vs. Smith Predictor. It is interesting to investigate the conditions under which the MKFP will reduce to the SP. If $\xi(t) \neq 0$, the MKFP will reduce to an SP when $G_F = 1$. Consider the filter G_F for the modified KFP (from Appendix C):

$$G_F(t, q^{-1}) = \frac{\Delta K_1(q^{-1}) + D(q^{-1})}{\Delta C(q^{-1}) + D(q^{-1})q^{-d}} \tag{30}$$

From this equation it is difficult to see whether G_F will be equal to unity. However, since $\Delta = 1 - q^{-1}$, at steady state it is easy to see that $G_{F_{ss}} = 1$.

Thus the MKFP reduces to a SP under steady state operation. This result also confirms the ability of the MKFP to handle deterministic disturbances as defined in the previous section.

Example. Figure 9 compares the response of a first-order system under PI control using the MKFP and the SP. Note that for this noise-free case, with no process mismatch, the responses to a set point change are identical, as proven earlier. For a step disturbance in load the response of the MKFP is better than the SP, but the key point is that the MKFP has eliminated the offset that results when using the basic KFP, Figure 5. Additional results are presented later.

Implementation of the KFP

The dimension of the specific state space formulation used in the KFP increases with the number of time delays. If the time-varying KF algorithm given in Eqs. 8 to 12 is used for the implementation of the KFP, there will be a large number of matrix operations, especially multiplications, which implies a heavy computational effort. As the time delay increases, the problem would become more severe.

The sparse nature of the matrix Φ, due to the canonical form used in the state space formulation of the KFP, can be used to obtain a more efficient algorithm (Walgama, 1986) that has significantly fewer multiplications to perform. If $n = 1$, $d = 3$, the direct implementation of the KFP using standard matrix operations needs 208 multiplications, whereas the simplified implementation needs only 12. This relatively small computational effort justifies the employment of the KFP.

Instead of implementing the KFP in state space form, it can be implemented in transfer function form, using the configuration obtained via the innovation model illustrated in Figure 6. This can be interpreted as an extension of the SP, by introducing a filter G_F. This does not reduce the computational effort compared to the simplified implementation, but gives a conceptually simpler implementation of the KFP.

Predictive Control

Since it is assumed that a process model is available for the design of the KFP, it makes sense to use the same model in the design of the feedback controller rather than use a conventional PID form. A predictive controller can be designed based on the estimated transfer function model of the process. For simplicity, since the output of the MKFP is a minimum variance estimate, $\hat{y}(\cdot)$, of the true plant output $\hat{y}(\cdot)$, the noise term is neglected in the design of the predictive controller. (This is equivalent to assuming $C = 1$ in the familiar ARMA model equation. Note that any controller design method can be used here.) Walgama (1986, ch. 2) derives two predictive controllers. Based on the ARMAX model (Eq. A.18, Appendix A) the predictive control law is,

$$u(t) = \frac{1}{b_1}\{y^*(t+d+1) - [1 - A(q^{-1})]\hat{y}(t+d+1) + [b_1 - B'(q^{-1})]u(t)\} \tag{31}$$

Using an ARIMA model (cf. Eqs. C.4 and 28) for the case where deterministic disturbances are present, the final result is an incremental control law:

$$\Delta u(t) = \frac{1}{b_1}[\delta y^*(t+d+1) - [1 - A(q^{-1})]\,\Delta\hat{y}(t+d|t) + [b_1 - B'(q^{-1})]\Delta u(t)] \tag{32}$$

where

$$\delta y^*(t+d+1) = y^*(t+d+1) - \hat{y}(t+d|t)$$
$$\Delta\hat{y}(t+d|t) = \hat{y}(t+d|t) - \hat{y}(t+d-1|t-1)$$
$$\Delta u(t) = u(t) - u(t-1)$$

Simulation Results and Discussion

Simulation studies were carried out for first- and second-order processes described by the following difference equations,

$$y(t) - 0.9321\,y(t-1) = -0.1717\,u(t-4) + 0.9329\,\xi(t)$$

$$y(t) - 1.8954\,y(t-1) + 0.8981\,y(t-2) = 0.7975\,u(t-4) - 0.7758\,u(t-5) + \xi(t)$$

In some runs white noise sequences were added to each process state (mean = 0.0 and variance = 0.001 for the first-order

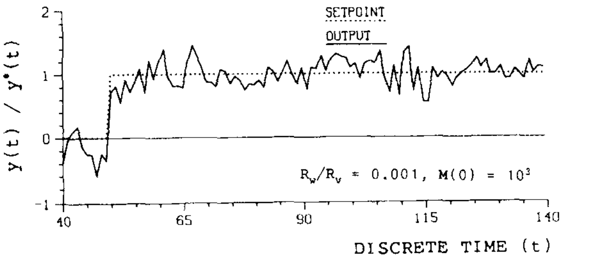

Figure 10. KFP plus predictive control of a second-order stochastic process.

model, and 0.0 and 0.1 for the second-order model) to generate the process noise, and to the output (0.0 and 0.1) to generate the measurement noise. The PI controller parameters (Meyer, 1977) were $K_p = -1.36$ and $\tau_I = 574$.

Perfect Modeling, No Noise. Under these conditions the performance of the KFP and the SP are identical, as suggested by the preceding theoretical analysis, and the use of a predictive controller gave perfect tracking of set point changes. The minimum variance controller of Astrom (1970) would give essentially the same performance.

Perfect Modeling, Stochastic Noise. Figure 3 shows the performance of the KFP with PI control of the first-order process. The performance of the Smith predictor in Figure 7 appears very similar, but as shown by Figure 4 actually had a higher variance ($\alpha = 1$ for no filter). As the exponential filter constant was decreased the performance of the SP became smoother but the variance increased. Since the KFP is optimal (for proper choice of the noise covariance matrices) the minimal variance in Figure 4 is lower than for any other filter.

The performance of the KFP and the SP for predictive control of the second-order process is illustrated by Figures 10 and 11. The KFP gives better control of the output $y(t)$ and also smoother manipulation of $u(t)$. As shown in Figure 12, the KFP can be tuned by varying the ratio R_w/R_v of the noise covariance matrices and the SP by varying the filter constant α. However the KFP still gives better performance.

Perfect Modeling with Disturbances. Disturbances $\xi(t)$ were introduced at the 100th time instant with amplitudes of 0.4 and 0.1 for the first- and second-order models, respectively. (Note that these are relatively large disturbances and without control would result in steady state outputs of 5.49 and 37.0, respectively). Figure 5 shows that the KFP with PI control gave significant offset that could be reduced only slightly by tuning R_w/R_v. Figure 9 shows that both the SP and modified KFP (with the

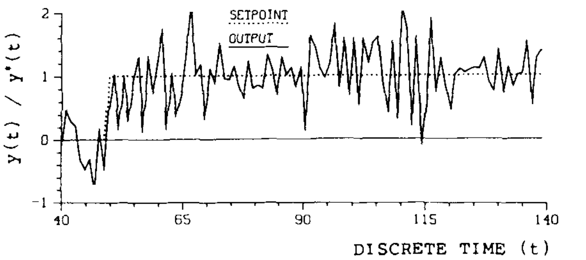

Figure 11. SP plus predictive control of a second-order stochastic process.

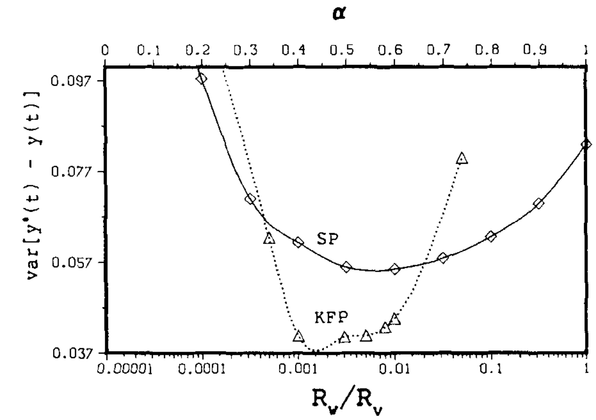

Figure 12. Output error variance for predictive control of a second-order process.
See Figures 10 and 11

same controller as in Figure 5) eliminated the offset. Figures 13 and 14 show significant improvement due to the use of the predictive controller on the first- and second- order processes, respectively (cf. Figures 5 and 9). Note again that the MKFP is significantly better than the SP and eliminates the offset problem observed with the KFP. The improved performance of the MKFP relative to the SP was traced to the fact that the MKFP predicts the disturbance as illustrated in Figure 15 for the second-order process; see Figure 14. This prediction is possible because the MKFP contains a model of the disturbances; Eq. 28 and 29.

Modeling Errors. For these runs the steady state gain of the first-order model was reduced by 20% compared to the actual process gain. As shown by Figures 16 and 17, the set point changes achieved by the SP and the MKFP were both significantly worse than the "perfect" set point changes in Figures 13 and 14. However, the MKFP gave significantly better performance than the SP. This is because the input to the filter (y_e in

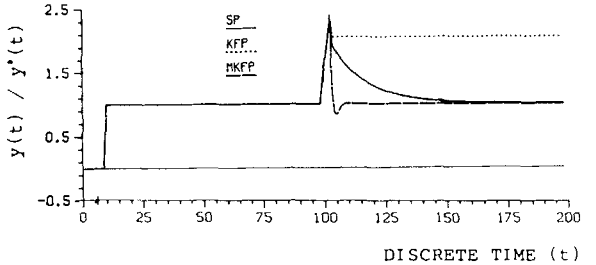

Figure 13. Predictive control of a first-order process.
See Figures 5 and 9

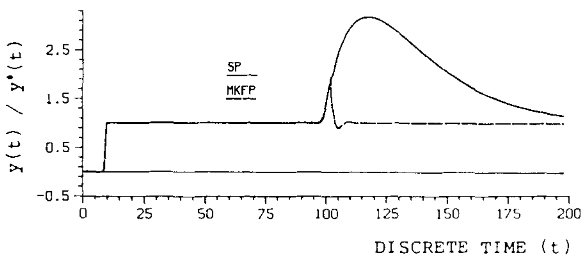

Figure 14. Predictive control of a second-order process.

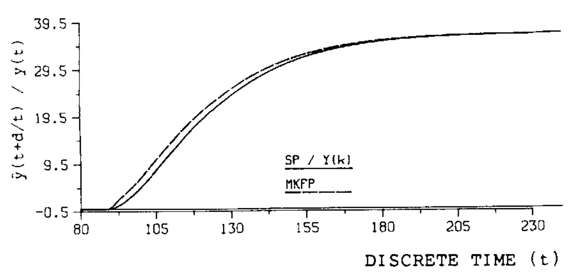

Figure 15. Open-loop process response showing prediction of disturbances by MKFP.

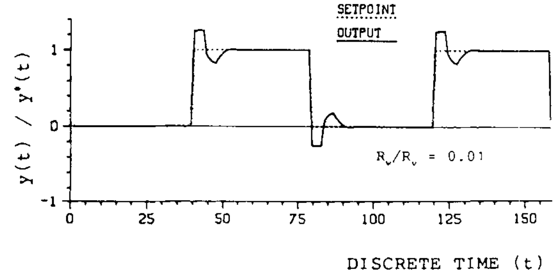

Figure 17. MKFP plus predictive control of a first-order process with gain mismatch.
See Figures 13 and 16

Figure 6) is equal to the disturbance $\xi(t)$ plus the effect of model/process mismatch and the filter can be tuned by varying R_w/R_v to give improved performance. (The KFP gave a small offset after the set point change.)

Overall Result. The MKFP gives significantly better performance than the SP, particularly for applications with noise plus sustained disturbances, and is recommended for practical applications. The approach used in this paper to develop a fixed-parameter MKFP can also be used to develop KFP's for multivariable processes with time delays or adaptive MKFP's for multiple-input/multiple-output processes.

Conclusions

1. The Kalman filter predictor is stable when applied to stable processes and gives minimum variance estimates of the future process output values, $\{y(t+i\,|\,t), i = 1, \ldots, d\}$, which can be used for time delay compensation or feedback control.

2. An innovation model analysis is used to convert the KFP from state space to an equivalent input/output form. It is shown that the KFP has the same block diagram structure as a Smith predictor except that it includes an optimal noise filter. The location of the filter is the same as in internal model control but is optimally designed to deal with process and measurement noise rather than being used to trade off stability vs. performance.

3. The KFP is modified to handle deterministic disturbances (e.g., steps) by including a model of the disturbance in the augmented state space model of the process. The modified KFP eliminates offset problems due to sustained disturbances and model/process mismatch. The modified KFP also "forecasts" the disturbance and when combined with a predictive feedback controller gives significantly better regulatory control than the Smith predictor.

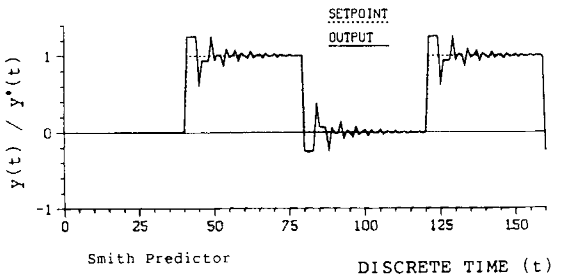

Figure 16. SP plus predictive control of a first-order process in the presence of 20% process gain mismatch.

Acknowledgment

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Notation

$A(q^{-1}), B(q^{-1})$ = ARMA model of process
$a_1 \ldots a_n$ = coefficients of $A(q^1)$ polynomial
$b_1 \ldots b_n$ = coefficient s of $B(q^{-1})$ polynomial
d = process time delay excluding zero-order hold
$E[\cdot]$ = statistical expectation of a random variable
G_c = transfer function of controller
$G(q^{-1})$ = transfer function of process
$G_p(q^{-1})$ = transfer function of process model without time delays
$G_m(q^{-1})$ = transfer function of process model including delay
$G_F(t,q^{-1})$ = transfer function of the filter in KFP
n = order of process
R_w = covariance of process noise
R_v = covariance of measurement noise
$u(t)$ = process input
x = $(n+d)$-order state vector of process with time delays
$y(t)$ = process output
$y_1(t)$ = output of process without time delays
$y^*(t)$ = desired output (set point)
$y(t+d/t)$ = prediction of future output at time $t+d$ based on data at time t

Greek letters

α = exponential filter constant
Δ = difference operator
$\xi(t)$ = process noise, measurement noise, and any type of disturbance to process

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